Review of Special Relativity from PS303

Here are some definitions and equations you should recall from PS303.

Kinematics: Since kinematics deals with space and time, this will require some changes to our classical understanding of distance (or displacement), as well as time intervals, as measured between inertial frames.

L = length $\quad$ L_o = "proper" length
$\tau =$ time interval $\quad$ $\tau_o =$ "proper" time interval

From the constancy of the speed of light between inertial frames (Einstein's 2nd postulate), we obtain the Lorentz transformations. From the Lorentz transformations we obtain the "length contractions" and "time dilations" measured between inertial frames

Lorentz Transformation:

$$x' = \gamma(x - \beta ct)$$

$$ct' = \gamma(ct - \beta x)$$

$L = \frac{L_o}{\gamma}$ and $\tau = \gamma\,\tau_o$ where $\gamma = \frac{1}{\sqrt{1-\beta^2}}$ and $\beta = \frac{V}{c}$

and $V =$ the velocity of the primed frame w.r.t. the unprimed frame

We also obtain the velocity transformation between inertial frames:

$$v'_x = \frac{v_x - V}{1 - \frac{v_x V}{c^2}}$$

Dynamics: Since dynamics includes mass, as well as space and time, this will require some changes to our classical understanding of Force, Momentum, and Energy. From Einstein's 1st postulate we obtain the mass "change" that occurs between inertial frames.

Momentum
γm = relativistic mass $\quad$ $m =$ "rest mass" $\quad$ momentum = $p = \gamma m\, v = \gamma m\ \beta c$

$$p = mc\,\beta\gamma$$

Energy
From the work-energy theorem we can show that: $\quad KE = mc^2(\gamma - 1)$

$E =$ total energy = kinetic energy (K) + potential energy (U) + "rest mass" energy (mc^2)

However, we will assume the potential energy is “zero” for now. So, $E = K + mc^2$. Another way to write the total energy is $E = \sqrt{p^2c^2 + m^2c^4}$.

Some useful shortcuts:

$$\beta = \frac{pc}{E}$$

$$\gamma = \frac{E}{mc^2}$$

$$K = mc^2(\gamma - 1)$$

$$E = \gamma mc^2$$

$$rest\ mass\ energy = mc^2$$